Week 8 Worksheet Thursday

Instructions. Follow the instructions given by your TA. You are not expected to finish all the problems. :)

Main Topic: Related Rates

1. A pole of length 10 feet rests against a vertical wall. If the $\underbrace{\text{bottom of the pole}}_{x}$ slides away from the wall at a speed of 2 ft/s, how fast is the $\underbrace{\text{angle}}_{\theta}$ between the top of the pole and the wall changing when the angle is $\frac{\pi}{4}$ radians?

x: length of bottom away from the wall

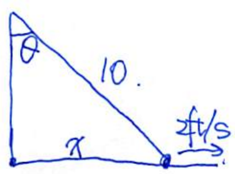
θ: angle between top and the wall.

$\sin\theta = \frac{x}{10}$

$\frac{dx}{dt} = 2$

$\theta = \frac{\pi}{4}$

$\frac{d\theta}{dt} = ?$



$$\cos\theta \frac{d\theta}{dt} = \frac{1}{10}\frac{dx}{dt}$$

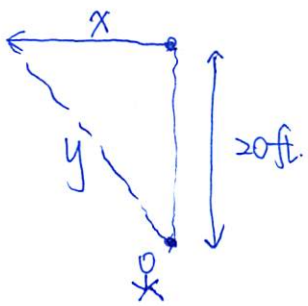
$$\frac{\sqrt{2}}{2}\frac{d\theta}{dt} = \frac{1}{10}\cdot 2$$

$$\frac{d\theta}{dt} = \frac{2}{5\sqrt{2}} = \frac{\sqrt{2}}{5} \text{ radians/second.}$$

2. A girl flies a kite at a height of 20ft, the wind carrying the kite horizontally away from her at a rate of 10ft per second. How fast must she let the string out when the kite is 40ft away from her?

x: distance of kite horizontally away from the girl $\quad \frac{dx}{dt} = 10$ ft/s

y: distance of kite away from the girl (string length) $\quad \frac{dy}{dt} = ? \quad y = 40$ft.



$$x^2 + 20^2 = y^2$$

$$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$

$\frac{dx}{dt} \to 10$, $y \to 40$, $\frac{dy}{dt} \to ?$

$x \to$ not directly given in the problem. but since $x^2 + 20^2 = y^2$, where $y = 40$

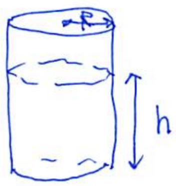
$$\Rightarrow x = \sqrt{40^2 - 20^2} = \sqrt{1200}$$

$$\Rightarrow \frac{dy}{dt} = \frac{2\cdot\sqrt{1200}\cdot 10}{2\cdot 40} = \frac{\sqrt{1200}}{4} = \frac{20\sqrt{3}}{4} = 5\sqrt{3} \text{ ft/s}$$

3. *(from 2013 midterm2)* A cylindrical container of radius R is being filled up with water at a rate of 10 cubic meters per hour. At what rate is the height of the water increasing? Your solution should appear as a function of R.

V: Volume of water

h: height of water



$$V = \pi R^2 h$$

$$\underbrace{\frac{dV}{dt}}_{=10} = \pi R^2 \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{10}{\pi R^2}$$

4. *(from HW7)* A spherical balloon is inflated with helium at the rate of 100π ft^3/min. How fast is the balloon's radius increasing at the instant the radius is 5 ft? How fast is the balloon's surface area increasing at the instant the radius is 5 ft?

① V: Volume of balloon $\qquad \frac{dV}{dt} = 100\pi$

r: radius of balloon $\qquad r = 5 \quad \frac{dr}{dt} = ?$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$100\pi = 4\pi\, 5^2 \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = 1 \quad \text{ft/s}.$$

② S: surface area of balloon

$$S = 4\pi r^2$$

$$\frac{dS}{dt} = 4\pi\, 2r \frac{dr}{dt}$$

$$\frac{dS}{dt} = 4\pi \cdot 2 \cdot 5 \cdot 1 = 40\pi \quad \text{ft}^2/\text{s}$$